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SHORTER COMMUNICATIONS

PRESSURE-INDUCED RADIATIVE TRANSFER IN HYDROGEN*

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INTRODUCTION

RECENTLY Cess and Khetan [1] have presented a modified differential approximation for describing radiative transfer due to the far infrared pressure-induced spectrum of hydrogen. The radiative approximation was developed for a variable-pressure system; i.e. the atmospheres of the major planets which are composed primarily of hydrogen. Results were presented for temperatures of 120°K and lower, consistent with temperature levels within the stratospheres of these planets.

In addition to planetary atmospheres, a terrestrial application of pressure-induced radiative transfer concerns heat transfer within compressed gas storage tanks. For example, Jones [2] has recently calculated total radiation properties for compressed oxygen. With this application in mind, the purpose of the present note is twofold. First, to extend the results of [1] to higher temperatures; and second, to rephrase the atmospheric formulation of [1] for application to constant-pressure systems with bounding surfaces.

RESULTS

We consider first the gas emissivity, which is defined as

$$\varepsilon(z) = (1/\sigma T^4)\int_0^\infty e_\omega(T)\,[1 - \exp(-\kappa_\omega z/P^2)]\,\mathrm{d}\omega \qquad (1)$$

where $z = P^2 y$ for uniform pressure, with y denoting the slab thickness, while ω, $e_\omega(T)$, σ and T denote wave number, Planck's function, the Stefan–Boltzmann constant, and temperature, respectively, and κ_ω is the spectral volumetric absorption coefficient. For pressure-induced absorption, $\kappa_\omega \sim P^2$.

At temperatures of present interest, the hydrogen opacity arises from rotational–translational and pure translational transitions. The calculation of κ_ω follows [1] and is similar to that of Trafton [3]. Additional calculations of κ_ω for hydrogen are given by Encrenaz *et al.* [4], Encrenaz [5] and Pollack [6]. The computed emissivities are illustrated in Fig. 1, and as for lower temperatures [1], an excellent approximation is of the form

$$\varepsilon(z) = \frac{a_0}{a_1}(1 - \mathrm{e}^{-a_1 z}). \qquad (2)$$

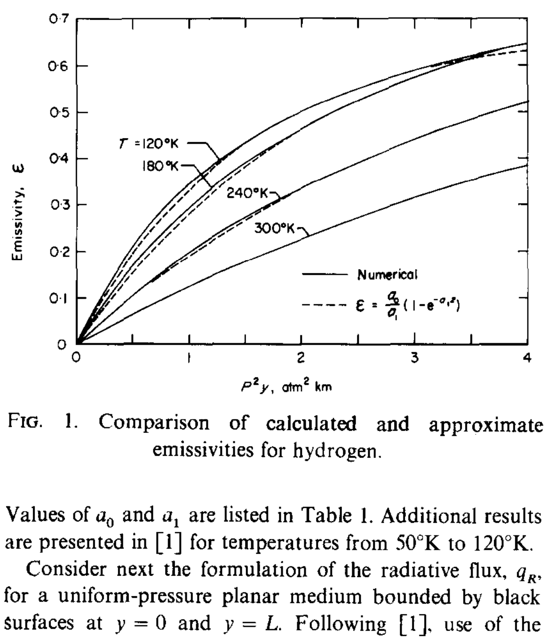

FIG. 1. Comparison of calculated and approximate emissivities for hydrogen.

Values of a_0 and a_1 are listed in Table 1. Additional results are presented in [1] for temperatures from 50°K to 120°K.

Consider next the formulation of the radiative flux, q_R, for a uniform-pressure planar medium bounded by black surfaces at $y = 0$ and $y = L$. Following [1], use of the exponential kernel approximation

$$E_2(x) \simeq b\,\mathrm{e}^{-cx} \qquad (3)$$

together with equation (2) allows the integrals in the radiative flux formulation to be eliminated in the same fashion as for a gray gas (Sparrow and Cess [7]). The application in [1]

Table 1. Values of a_0 and a_1

Temperature (°K)	120	140	160	180	200	220	240	260	280	300
a_0 atm^{-2} km^{-1}	0·46	0·43	0·38	0·35	0·30	0·26	0·22	0·19	0·16	0·14
a_1 atm^{-2} km^{-1}	0·68	0·56	0·54	0·45	0·40	0·34	0·29	0·25	0·22	0·19

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was to a deep atmosphere with no far infrared irradiation at the top of the atmosphere. The only modification required in the present situation concerns inclusion of the surface emission terms. If we let $\tau = a_1 P^2 y$ denote an optical coordinate, with $\tau_0 = a_1 P^2 L$ the corresponding optical thickness, the present formulation is found to be identical to that for a gray gas providing q_R in the gray gas analysis is replaced by q_R^*, where

$$q_R^* = \frac{a_1}{a_0}\left[q_R - \left(1 - \frac{a_0}{a_1}\right)\sigma(T_1^4 - T_2^4)\right]. \qquad (4)$$

The above procedure is straightforward, and ideally one would like the approximate formulation, in terms of the equivalent gray gas expression $q_R^*(\tau)$, to reduce to the correct optically thick limit, as well as producing the correct expressions for both q_R^* and $\mathrm{d}q_R^*/\mathrm{d}\tau$ in the optically thin limit. This is not possible, however, with a single choice for b and c in equation (3). Thus, instead of following the procedure employed in [1], we will simply modify the conventional differential approximation (or Eddington approximation), as applied to a gray gas, through replacing q_R by q_R^*. This circumvents the need for equation (3), and from Sparrow and Cess [7], or Goody [8], q_R^* is described by

$$\frac{\mathrm{d}^2 q_R^*}{\mathrm{d}\tau^2} - 3q_R^* = 4\sigma\frac{\mathrm{d}T^4}{\mathrm{d}\tau} \qquad (5)$$

with the boundary conditions

$$\tfrac{1}{2}q_R^*(0) - \tfrac{1}{4}\left(\frac{\mathrm{d}q_R^*}{\mathrm{d}\tau}\right)_{\tau=0} = \sigma T_1^4 - \sigma T^4(0) \qquad (6a)$$

$$\tfrac{1}{2}q_R^*(\tau_0) + \tfrac{1}{4}\left(\frac{\mathrm{d}q_R^*}{\mathrm{d}\tau}\right)_{\tau=\tau_0} = \sigma T^4(\tau_0) - \sigma T_2^4 \qquad (6b)$$

where T_1 and T_2 are the $y = 0$ and $y = L$ surface temperatures, respectively.

In summary, q_R^* constitutes an equivalent gray gas radiative flux, and the above equations are precisely those for a gray gas. Equation (4) in turn converts q_R^* to the actual radiative flux, q_R, for pressure-induced radiation.

A similar procedure for treating real-gas radiative transfer has been proposed by Gilles, Cogley and Vincenti [9]. The only basic difference is that they employ a linearization, such that in present nomenclature the radiative flux is expressed in terms of the modified emissivity, for which the weighting function is the temperature derivative of Planck's function (Wang [10]). With respect to atmospheric applications, however, the linearization is inconvenient, since both the emissivity and modified emissivity enter into the radiative flux formulation.

The present procedure, when applied to planetary (Cess and Khetan [1]) and satellite (Cess and Owen [11]) hydrogen atmospheres, yields good agreement with more detailed numerical calculations. Comparable agreement should exist for uniform-pressure systems.

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